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## ABSTRACT

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# A Fixed Effects Panel Data Model: Mathematics Achievement in the U.S.

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## ABSTRACT

Statistical models that combine cross section and time series data offer analysis and interpretation advantages over separate cross section or time series data analyses (Matyas & Severstre, 1996). Data that embodies time series and cross section units have not been commonplace in the research community until the last 25 years (Tieslau, 1999). In this study, a fixed effects panel data model is applied to the National Education Longitudinal Study of 1988 (NELS:88) data to determine if educational process variables, teacher emphasis, student self-concept, and socio-economic status can account for variance in student mathematical achievement. A model that includes seven independent variables accounted for 25% of the variance in student mathematical achievement test score. The primary objective of this study was not to explain mathematics achievement, but provide educational researchers with a functional and viable model for panel data analyses.

**A Fixed Effects Panel Data Model:  
Mathematics Achievement in the U.S.**

Data that embodies time series and cross section units have not been commonplace in the research community until the last 25 years (Tieslau, 1999). In fact, the U.S. Department of Education's National Center for Education Statistics (NCES) was not mandated to "collect and disseminate statistics and other data related to education in the United States" until the Education Amendments of 1974 (Public Law 93-380, Title V, Section 501, amending Part A of the General Education Provisions Act). Researchers commonly have termed data that contains time series and cross section units to be panel or longitudinal data. In this study, these terms are used interchangeably. Essentially, panel data is a set of individuals who are repeatedly sampled at different intervals in time, across a multitude of cross sectional variables. The term "individual" might be used loosely to imply a person, a household, a school, school districts, firms, or a geographical region. Figure 1 provides a typical structure for a panel data set. Schools have been used to represent the different "individuals". (Note: the individual unit could just as well have been different schools within a particular district, school districts within a state, or an aggregate representation by state).

Researchers who are interested in understanding, explaining, or predicting variation within longitudinal data are faced with some complex stochastic specifications. The problem that occurs when realizations exhibit two-dimensional variation—variation across time and cross section, is model specification. In other words, researchers need to specify a model that can capture individual differences in behavior across individuals and/or through time for estimation and inference purposes (Greene, 1997).

$$Y_{it} = \begin{bmatrix} Y_{\text{school}\#1, \text{year}\#1} \\ Y_{\text{school}\#1, \text{year}\#2} \\ \vdots \\ Y_{\text{school}\#1, \text{year}\#T} \\ \dots \\ Y_{\text{school}\#2, \text{year}\#1} \\ Y_{\text{school}\#2, \text{year}\#2} \\ \vdots \\ Y_{\text{school}\#2, \text{year}\#T} \\ \dots \\ Y_{\text{school}\#N, \text{year}\#1} \\ Y_{\text{school}\#N, \text{year}\#2} \\ \vdots \\ Y_{\text{school}\#N, \text{year}\#T} \end{bmatrix} \quad X_{it} = \begin{bmatrix} X_{1\text{school}\#1, \text{year}\#1} & X_{2\text{school}\#1, \text{year}\#1} & \dots & X_{K\text{school}\#1, \text{year}\#1} \\ X_{1\text{school}\#1, \text{year}\#2} & X_{2\text{school}\#1, \text{year}\#2} & \dots & X_{K\text{school}\#1, \text{year}\#2} \\ \vdots & \vdots & & \vdots \\ X_{1\text{school}\#1, \text{year}\#T} & X_{2\text{school}\#1, \text{year}\#T} & \dots & X_{K\text{school}\#1, \text{year}\#T} \\ \dots & \dots & \dots & \dots \\ X_{1\text{school}\#2, \text{year}\#1} & X_{2\text{school}\#2, \text{year}\#1} & \dots & X_{K\text{school}\#2, \text{year}\#1} \\ X_{1\text{school}\#2, \text{year}\#2} & X_{2\text{school}\#2, \text{year}\#2} & \dots & X_{K\text{school}\#2, \text{year}\#2} \\ \vdots & \vdots & & \vdots \\ X_{1\text{school}\#2, \text{year}\#T} & X_{2\text{school}\#2, \text{year}\#T} & \dots & X_{K\text{school}\#2, \text{year}\#T} \\ \dots & \dots & \dots & \dots \\ X_{1\text{school}\#N, \text{year}\#1} & X_{2\text{school}\#N, \text{year}\#1} & \dots & X_{K\text{school}\#N, \text{year}\#1} \\ X_{1\text{school}\#N, \text{year}\#2} & X_{2\text{school}\#N, \text{year}\#2} & \dots & X_{K\text{school}\#N, \text{year}\#2} \\ \vdots & \vdots & & \vdots \\ X_{1\text{school}\#N, \text{year}\#T} & X_{2\text{school}\#N, \text{year}\#T} & \dots & X_{K\text{school}\#N, \text{year}\#T} \end{bmatrix}$$

Figure 1. Panel Data Structure

In general, longitudinal (panel) data sets contain a large number of cross-section units and a relatively small number of time-series units.

The U.S. Department of Education began collecting data in 1988 about critical transitions experienced by students as they leave elementary schools and progress through high school and into postsecondary institutions or the work force. The National Education Longitudinal Study of 1988 (NELS:88) contains data about educational processes and outcomes pertaining to student learning, predictors of dropping out, and school effects on students' access to programs and equal opportunities to learn. The first follow-up was conducted with the same students, their teachers, and principals in 1990. The second follow up survey was conducted in 1992, and the third in 1994. Data from NELS:88 will be used in this study to determine if student perception of educational process variables can account for the variance in mathematical achievement.

### Model Specification

When should a fixed effects or random effects model be utilized? The answer to this question is often debated. Some believe that it is dependent upon the underlying cause in the

model. For example, if the individual effects are the result of a large number of non-observable stochastic variables, then the random effect interpretation is demanded. Others think the decision rest on the nature of the sample – that is when the sample is comprehensive or exhaustive, then fixed effects models is the natural choice to enhance the generalizability. On the contrary, if the sample does not contain a large percent of the population then the random effect model would be the model of choice. According to Hsiao (1985), it is ultimately, “up to the investigator to decide whether he wants to make an inference with respect to population characteristics or only with respect to effects that are in the sample” (pg. 131). It is unlikely that this debate will ever be resolved per se, however, if the choice between the two underlying causes is clear, then the estimation method should be chosen accordingly. However, if the choice is not clear, then the decision should be based on the nature of the sample and statistical evidence. For example, if the t-stats on the individual effects are mostly significant then this is clearly a sign that a significant component of the model is accounted for by the individual effect parameter and so FE might be preferred to RE. However, if the Hausman test statistic, a statistic designed to test model fit, is large then one might have reason to believe that a RE model is preferred.

### Fixed Effects Models

The fixed effects (FE) model takes  $\alpha_i$  to be a individual specific constant term in the regression equation

$$Y_{it} = \alpha_i + \beta_1 X_{1it} + \beta_2 X_{2it} + \dots + \beta_K X_{Kit} + \varepsilon_{it} \quad (1)$$

or in matrix notation

$$Y_i = \alpha_i + X_i' \beta + \varepsilon_{it} \quad (2)$$

where

$$X_{it}' = [X1_{it} \ X2_{it} \dots \ XK_{it}] \quad \text{and} \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}.$$

The “ $i$ ” indexes cross-section realizations so that  $i = 1, 2, 3, \dots, N$  and “ $t$ ” indexes time-series realizations so that  $t = 1, 2, 3, \dots, T$ . The *individual effect*  $\alpha_i$ , is regarding to be constant over time ( $t$ ) and specific to the individual cross-sectional unit ( $i$ ). The term  $\alpha_i$  is presumed to captures the unobservable, and non-measurable characteristics that differentiate individual units. Basically, this implies that all behavioral differences between individuals (e.g., schools in Figure 1) are fixed over time and are represented as parametric shifts of the regression function. Mátyás and Sevestre (1996, p. 34) state, “the intercept is allowed to vary from individual to individual while the slope parameters are assumed to be constants in both the individual and time dimensions”. The fundamental assumptions of the fixed effects model are:

- (1)  $E[\varepsilon_{it}] = 0$ ,
- (2)  $\text{cov}(\varepsilon_{it}, \varepsilon_{jt}) = 0$ ,
- (3)  $\text{var}(\varepsilon_{it}) = E[\varepsilon_{it}^2] = \sigma_e^2$ ,
- (4)  $E[\varepsilon_{it}, X1_{it}] = E[\varepsilon_{it}, X2_{it}] = E[\varepsilon_{it}, X3_{it}] = \dots = E[\varepsilon_{it}, XK_{it}] = 0$ , and
- (5)  $X_{it}$  is not invariant.

Under these assumptions, the original least squared estimator (OLS) can be used to obtain unbiased, consistent, and efficient (BLUE) parameter estimates.

### Random Effects Model

The random effects (RE) model—also known as the error component model, includes a non-measurable stochastic variable, which differentiates individuals. It is written as:

$$Y_{it} = \alpha + \beta_1 X_{1it} + \beta_2 X_{2it} + \dots + \beta_K X_{Kit} + u_i + \varepsilon_{it}, \quad (3)$$

or in matrix notation:

$$Y_i = \alpha + X_{it}' \beta + u_i + \varepsilon_{it}, \quad (4)$$

where:  $X_{it}' = [X_{1it} \ X_{2it} \ \dots \ X_{Kit}]$  and  $\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}$ .

The “ $i$ ” indexes cross-section realizations so that  $i=1,2,3,\dots, N$  and “ $t$ ” indexes time-series realizations so that  $t = 1,2,3,\dots, T$ . The term “ $u_i$ ” is a stochastic variable that embodies the *unobservable* or *non-measurable* disturbances that accounts for individual differences.

Essentially, the effect is thought to be a random individual effect rather than a fixed parameter.

For example, a researcher might try to discern whether there is a difference in achievement between districts in the State of Texas. Instead of including every school district in the equation (as we would have in the fixed effects model using dummy variables) one can randomly sample school districts and assume that the effect is random distributed across “individuals” but is constant through time.



The fundamental assumptions of the random effects model are as such:

$$E[u_i, X1_i] = E[u_i, X2_i] = E[u_i, X3_i] = \dots = E[u_i, XK_i] = 0,$$

$$E[\varepsilon_{it}] = E[u_i] = 0,$$

$$\text{var}(\varepsilon_{it}) = E[\varepsilon_{it}^2] = \sigma_e^2,$$

$$\text{var}(u_i) = E[u_i^2] = \sigma_u^2,$$

$$\text{cov}(u_i, \varepsilon_{it}) = E[u_i, \varepsilon_{it}] = \sigma_{\varepsilon, u}.$$

Assuming normality  $u_i \sim N(0, \sigma_u^2)$ ,  $\varepsilon_{it} \sim N(0, \sigma_e^2)$ , both “ $u_i$ ” and “ $\varepsilon_{it}$ ” are stochastic

variables, but form one composite error term-called omega ( $u_i + \varepsilon_{it} \equiv \omega_{it}$ ).

$$\text{Where } \omega_{it} = \begin{bmatrix} u_1 & + & \varepsilon_{11} \\ u_1 & + & \varepsilon_{12} \\ & \vdots & \\ u_1 & + & \varepsilon_{1T} \\ \dots & \dots & \dots \\ u_2 & + & \varepsilon_{21} \\ u_2 & + & \varepsilon_{22} \\ & \vdots & \\ u_2 & + & \varepsilon_{2T} \\ \dots & \dots & \dots \\ u_N & + & \varepsilon_{N1} \\ u_N & + & \varepsilon_{N2} \\ & \vdots & \\ u_N & + & \varepsilon_{NT} \end{bmatrix}$$

The error term now consist of two components: (1) the error disturbance  $\varepsilon_{it}$ , and (2) the individual specific disturbance  $u_i$ . The RE model now takes the form of

$$Y_{it} = \alpha + \beta_1 X_{1it} + \beta_2 X_{2it} + \dots + \beta_K X_{Kit} + \omega_{it}, \quad (5)$$

or in matrix notation  $Y_i = \alpha + X_i' \beta + \omega_i$ .

$$(6)$$

The error term in the model now exhibits the following characteristics:

$$\text{var}(\omega_{it}) = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon, u} \\ \sigma_{u, \varepsilon} & \sigma_u^2 \end{bmatrix}.$$

As it stands, the original least squares estimator may not be applied to equation 6 because the error term not longer possess ideal properties (constant variance and zero covariance) thus the estimate would be inefficient and, hence generalized least squares (GLS) is appropriate.

However, the nature of data in behavioral sciences does not permit the variance components

$\sigma_u^2$  and  $\sigma_\varepsilon^2$  to be known, therefore, alternative estimation methods must be utilized. One

common estimation method that can deal with the unknown variance components feasible

generalized least squares (FGLS). Essential, FGLS takes an estimate of the variance components

and then estimates the equation.

The individual effect in the random effect model may, too, be tested with the following

hypotheses:  $H_o : u_i = 0$  or, equivalent ly,  $\sigma_u^2 = 0$   
 $H_a : \sigma_u^2 \neq 0$ .

After correcting the error term ( $\omega_{it}$ ) the t and F-test are reliable, thus inference can be regarded as valid. Base on statistical evidence, a FE model will be use estimated and will discussed later in the this paper..

#### National Education Longitudinal Data Set:88

The NELS:88 database is divided into two sections: (1) N2P, and (2) N4P. In this study, data were extracted from N2P. A representative sample of students (N=16,749) enrolled in tenth grade in the spring of 1990 who completed a questionnaire in both the first follow-up and second follow-up were identified and used in the analysis.

Seven independent variables are included in the specified model. They are listed as entered into the model: (1) Review Work (F2S19BA), (2) ListenLecture (F2S19BB), (3) CopyNotes (F2S19BC), (4) Calculators (F2S19BF), (5) Think Problem (F2S20D), (6) SES (F2SES1), and (7) Self concept (F2CNCPT). The first four variables, (ReviewWork, ListenLecture, Copynotes, Calculators), are frequency measures of student educational processes and are scaled as followed: (1) Never/Rarely, (2) 1-2 Times/Month, (3) 1-2 Times/Week, (4) Almost each day, (5) Every Day. For example, the variable ReviewWork is a measure of how frequently students review their work for the previous day. The variable ListenLecture is a measure of how frequently students listen to the teacher's lecture. The variable CopyNotes is a measure of how frequently students take notes. The variable Calculator is a measure of how frequently students use calculators. The variable ThinkProblem measures student perception of teachers emphases on mathematical objectives and is scaled, (0) none, (1) minor emphasis, (2) moderate emphasis, (3) major emphasis. SES is a continuous variable indicating socioeconomic status. This measure is based on Duncan's 1961 socioeconomic index for all occupations. It was derived from the parent questionnaire data, the student questionnaire data, or the first follow-up

or second follow-up New Student Supplement data. This variable has been standardized to have a mean of 0 and standard deviation of 1.

Selfconcept is a composite measure of all of the self-concept items (question 66) in the student questionnaire. Essentially, this variable measures students' self concept on a four point scale with: (1) strongly agree ,(2) agree ,(3) disagree,(4) strongly disagree. It should be noted that this variable was reversed scaled before composite score was created and was standardized to have mean of zero and a standard deviation of 1. MathScore is the dependent variable and it was derived by Item response theory (IRT) to have a mean of 50 and standard deviation of 10.

### Empirical Results

The FE model below was specified and estimated.

$$(\text{Mathscore}_{ii}) = \beta_1 + \beta_2(\text{Re viewWork})_{ii} + \beta_3(\text{ListenLecture})_{ii} + \beta_4(\text{CopyNotes})_{ii} + \beta_5(\text{Calculators})_{ii} \\ + \beta_6(\text{Think Pr oblem})_{ii} + \beta_7(\text{SES})_{ii} + \beta_8(\text{Selfconcept})_{ii} + \varepsilon_{ii}$$

Table 1 provides evidence that the FE model is indeed the correct choice over the RE model; note all t-stats are significant. Region is the cross section unit ( $i = 1,2,3,4$ ) indicating which of the four US Census regions (1) Northeast,(2) Midwest,(3) South, or (4)West.

Table1  
Estimated Fixed Effects

Group	Coefficient	Standard Error	t-ratio
North East	45.84312*	0.42968	106.69106
North Central	45.48305*	0.44005	103.35884
South	43.06207*	0.42588	101.11422
West	44.10279*	0.44472	99.16970

\*Statistically significant at the 99% level.

Table 2 provides the descriptive statistics, measure of central tendency, measure of dispersion, minimum and maximum, and number of realizations. Table 3 provides the correlation coefficients for all of the variables used in the analysis.

Table2  
Descriptive Statistics

Variable	Mean	Std.Dev.	Minimum	Maximum	Cases
ReviewWork	3.87	1.21	1.00	5.00	13577
ListenLecture	4.24	1.03	1.00	5.00	13565
CopyNotes	4.01	1.27	1.00	5.00	13567
Calculators	3.69	1.38	1.00	5.00	13560
ThinkProblem	2.23	0.84	0.00	3.00	13568
SES	0.04	0.81	-3.24	2.75	16563
Selfconcept	0.01	0.70	-3.69	1.24	15123
MathScore	51.81	9.93	29.50	71.49	12992
Region	2.56	1.01	1.00	4.00	16426

Table 3

## Correlation Matrix

	Math Score	Review Work	Listen Lecture	Copy Notes	Calculators	Think Problem	SES	Self Concept	Region
Math Score	1.000								
Review Work	.099	1.000							
Listen Lecture	.119	.369	1.000						
Copy Notes	.103	.278	.535*	1.000					
Calcu- lators	.148	.135	.140	.095	1.000				
Think Problem	.181	.224	.256	.237	.099	1.000			
SES	.470	.042	.044	.084	.103	.050	1.000		
Self Concept	.144	.085	.085	.076	.053	.134	.081	1.000	
Region	-.103	.009	.022	-.031	.074	-.009	-.082	.022	1.000

Note the moderate correlation between the two exogenous variables, CopyNotes and ListenLecture ( $\rho = .540$ ) is indicative of multicollinearity. This correlation gives reason to question the inference drawn from the t-stats on these two variables. However, the parameter estimates for CopyNotes and ListenLecture are still BLUE.

Six out of the seven independent variables were statistically significant at the 0.000 alpha level accounting for 25% of the variance in the dependent variable (mathematics achievement score).

See Table 4 for parameter estimates.

Table 4  
Fixed-Effects Estimates

Variable	Coefficient	Standard Error	b/St.Er	P[ Z >z]
ReviewWork	0.2342*	0.0716	3.272	0.0011
ListenLecture	0.3704*	0.0944	3.924	0.0001
CopyNotes	-0.1090	0.0732	-1.488	0.1367
Calculators	0.6669*	0.6007	11.100	0.0000
ThinkProblem	1.3877*	0.0970	14.304	0.0000
SES	4.9379*	0.1034	47.733	0.0000
SelfConcept	1.1370*	0.1129	10.071	0.0000
R-Square	.246			
Adjust R-Square	.245			

\*Statistically significant at 99% level.

The variable ReviewWork is statistically significant ( $t = 3.27$ ,  $p > .001$ ). As ReviewWork increases by one unit, MathScore increases by 0.234 points, c.p.. In other words, as students increase the frequency in which they review their work, holding all else constant, their math score increases by 0.234 points.

The variable ListenLecture is statistically significant ( $t = 3.924$ ,  $p > .000$ ). As ListenLecture increases by one unit, MathScore increases by 0.370 points. Or put differently, the

more attentive the student is to the teacher's lecture, their math score increases by 0.370 points, c.p.. The variable CopyNotes is not statistically significant ( $t = -1.488$ ,  $p > .1367$ ).

The variable Calculators is statistically significant ( $t = 11.10$ ,  $p > .000$ ). As Calculators increases by one unit, MathScore increase by 0.667 points, c.p.. Essentially, this estimate is showing that students math score will increase with the use of a calculator.

The variable ThinkProblem is statistically significantly ( $t = 14.304$ ,  $p > .000$ ). Recall, this variable measures student perception of teachers emphases on mathematical objectives and is scaled, (0) none, (1) minor emphasis, (2) moderate emphasis, (3) major emphasis. As ThinkProblem increases by 1 unit, MathScore increases by 1.387 point. Abstracted differently, the more teachers emphasis "thinking about what a problem means and ways it might be solve", holding all else constant, students math score increase by 1.387 points.

The variable SES is statistically significant ( $t = 47.333$ ,  $p > 0.000$ ). As SES increases by 1 unit, holding all else constant, MathScore increases by 4.937 points. Recall SES is a continuous variable indicating sample member's socioeconomic status. This measure is based on Duncan's 1961 socioeconomic Index for all Occupations.

The variable SelfConcept is statistically significant ( $t = 10.071$ ,  $p > 0.000$ ). As SelfConcept increases by 1 unit, MathScore increases by 1.137 points, c.p.. Essentially, students who have a more positive self perception, are scoring higher on the standardized math test.

### Conclusions

In this study, a fixed effects panel data model were applied to the National Education Longitudinal Study of 1988 (NELS:88). The empirical evidence presented here suggests that student mathematics test score is influence by educational process variables, teacher emphasis, student self-concept, and socio-economic status. Specifically, a model that includes seven

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independent variables accounted for 25% of the variance in student mathematical achievement test score. The primary objective of this study was not to explain mathematics achievement, but provide educational researchers with a functional and viable model for panel data analyses.

### Recommendation

The NELS:88 data set does not have a means of extracting the time component of the data. Although, the models for analyzing time effects were not discussed in the study, it is an important aspect of panel data that should be code when the data file is constructed. In addition, the time series unit should be measured in smaller periods of time. Further analysis should be performed on the error term ( $\epsilon_{it}$ ). That is, the error term should be analyzed for hetroskedascity and autocorrelation and problems.

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## Appendix

### LimDep Code

```

READ;File=C:\WINDOWS\Program Files\ES Limdep\PROGRAM\nels6.lpj;
  Nobs=16749;
  Nvar=23;
  Names=x1,x2,x3,x4,x5,x6,x7,x8,x9,x10,x11,x12,x13,x14,x15,x16,x17,x18,
  x19,x20,x21,x22,x23$

SKIP$

DSTATS; RHS = X2,X3,X4,X7,X13,X19,X20,X23; OUTPUT = 2 $

REGRESS;Lhs=X22
      ;Rhs=X2,X3,X4,X7,X13,X19,X20
      ;Str=X23
      ;Wts=X16
      ;Panel $

REGRESS;Lhs=X22
      ;Rhs=X2,X3,X4,X7,X13,X19,X20
      ;Str=X23
      ;Panel
      ;Output=2
      ;Wts=X16
      ;Fixed $

```

### Data Output

```

--> SKIP$
--> DSTATS; RHS = X2,X3,X4,X7,X13,X19,X20,X23; OUTPUT = 2 $

```

#### Descriptive Statistics

All results based on nonmissing observations.

Variable	Mean	Std.Dev.	Minimum	Maximum	Cases
X2	3.87029535	1.21401018	1.00000000	5.00000000	13577
X3	4.24061924	1.02966250	1.00000000	5.00000000	13565
X4	4.00906612	1.27206705	1.00000000	5.00000000	13567
X7	3.69041298	1.37809060	1.00000000	5.00000000	13560
X13	2.23349057	.839799327	.000000000	3.00000000	13568
X19	.485648735E-01	.811172698	-3.24000000	2.75000000	16563
X20	.111955300E-01	.701701693	-3.69000000	1.24000000	15123
X23	2.55777426	1.01482683	1.00000000	4.00000000	16426

Correlation Matrix for Listed Variables

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## Fixed Effects Panel Data Model 19

	X2	X3	X4	X7	X13	X19	X20	X23
X2	1.00000	.38402	.28129	.13514	.22252	.04671	.08881	.00758
X3	.38402	1.00000	.54066	.13636	.26014	.04523	.08992	.01662
X4	.28129	.54066	1.00000	.09404	.23307	.07737	.07659	-.03424
X7	.13514	.13636	.09404	1.00000	.10102	.10769	.05500	.07762
X13	.22252	.26014	.23307	.10102	1.00000	.05100	.14062	-.01871
X19	.04671	.04523	.07737	.10769	.05100	1.00000	.08907	-.08566
X20	.08881	.08992	.07659	.05500	.14062	.08907	1.00000	.01745
X23	.00758	.01662	-.03424	.07762	-.01871	-.08566	.01745	1.00000

```
--> REGRESS;Lhs=X22
      ;Rhs=X2,X3,X4,X7,X13,X19,X20
      ;Str=X23
      ;Wts=X16
      ;Panel $
```

```
+-----+
| OLS Without Group Dummy Variables
| Ordinary least squares regression Weighting variable = X16
| Dep. var. = X22 Mean= 52.55493863 , S.D.= 9.343603189
| Model size: Observations = 10895, Parameters = 8, Deg.Fr.= 10887
| Residuals: Sum of squares= 731652.8662 , Std.Dev.= 8.19782
| Fit: R-squared= .230712, Adjusted R-squared = .23022
| Model test: F[ 7, 10887] = 46 6.44, Prob value = .00000
| Diagnostic: Log-L = -38376.9800, Restricted(b=0) Log -L = -39805.8044
| LogAmemiyaPrCrt.= 4.208, Akaike Info. Crt.= 7.046
| Panel Data Analysis of X22 [ONE way]
| Unconditional ANOVA (No regressors)
| Source Variation Deg. Free. Mean Square
| Between 20751.0 3. 6917.00
| Residual 930327. 10891. 85.4216
| Total 951078. 10894. 87.3029
+-----+
```

```
+-----+
| Variable | Coefficient | Standard Error | b/St.Er. | P[|Z|>z] | Mean of X |
+-----+
| X2 | .2150986215 | .72295926E -01 | 2.975 | .0029 | 3.8895953 |
| X3 | .3746630995 | .95235646E -01 | 3.934 | .0001 | 4.2207458 |
| X4 | -.1516385665 | .73563678E -01 | -2.061 | .0393 | 3.9716586 |
| X7 | .7025006784 | .59425942E -01 | 11.821 | .0000 | 3.7239903 |
| X13 | 1.398697309 | .97973819E -01 | 14.276 | .0000 | 2.2173183 |
| X19 | 5.040076855 | .10413200 | 48.401 | .0000 | .86701895E -01 |
| X20 | 1.035652953 | .11377009 | 9.103 | .0000 | .42022957E -01 |
| Constant | 44.54122268 | .41506161 | 107.312 | .0000 |
```

```
+-----+
| Least Squares with Group Dummy Variables
| Ordinary least squares regression Weighting variable = X16
| Dep. var. = X22 Mean= 52.55493863 , S.D.= 9.343603189
| Model size: Observations = 10895, Parameters = 11, Deg.Fr.= 10884
| Residuals: Sum of squares= 716924.2581 , Std.Dev.= 8.11601
| Fit: R-squared= .246198, Adjusted R-squared = .24551
| Model test: F[ 10, 10884] = 355.48, Prob value = .00000
| Diagnostic: Log-L = -38266.1998, Restricted(b=0) Log -L = -39805.8044
| LogAmemiyaPrCrt.= 4.189, Akaike Info. Crt.= 7.027
| Estd. Autocorrelation of e(i,t) -.000540
+-----+
```

## Fixed Effects Panel Data Model 20

Variable	Coefficient	Standard Error	b/St.Er.	P[ Z >z]	Mean of X
X2	.2342486655	.71596894E -01	3.272	.0011	3.8895953
X3	.3704126069	.94402933E -01	3.924	.0001	4.2207458
X4	-.1089542352	.73215840E -01	-1.488	.1367	3.9716586
X7	.6668605394	.60076332E -01	11.100	.0000	3.7239903
X13	1.387686017	.97011428E -01	14.304	.0000	2.2173183
X19	4.937094032	.10343209	47.733	.0000	.86701895E -01
X20	1.136968803	.11289363	10.071	.0000	.42022957E -01

## Test Statistics for the Classical Model

Model	Log -Likelihood	Sum of Squares	R -squared
(1) Constant term only	-39805.80420	.9510780165D+06	.0000000
(2) Group effects only	-39685.63264	.9303270109D+06	.0218184
(3) X - variables only	-38376.97986	.7316528662D+06	.2307120
(4) X and group effects	-38266.19962	.7169242581D+06	.2461983

## Hypothesis Tests

Likelihood Ratio Test				F Tests		
	Chi-squared	d.f.	Prob.	F	num. denom.	Prob value
(2) vs (1)	240.343	3	.00000	80.975	3 10891	.00000
(3) vs (1)	2857.649	7	.00000	466.435	7 10887	.00000
(4) vs (1)	3079.209	10	.00000	355.481	10 10884	.00000
(4) vs (2)	2838.866	7	.00000	462.825	7 10884	.00000
(4) vs (3)	221.560	3	.00000	74.534	3 10884	.00000

REGR;PANEL. Could not invert VC matrix for Hausman test.

```

Random Effects Model: v( i,t) = e(i,t) + u(i)
Estimates: Var[e]           = .658696D+02
            Var[u]           = .489455D+01
            Corr[v(i,t),v(i,s)] = .069167
Lagrange Multiplier Test vs. Model (3) = 6971.10
( 1 df, prob value = .000000)
(High values of LM favor FEM/REM over CR model.)
Fixed vs. Random Effects (Hausman)      = .23
( 7 df, prob value = 1.000000)
(High (low) values of H favor FEM (REM).)
Reestimated using GLS coefficients:
Estimates: Var[e]           = .663837D+02
            Var[u]           = .499445D+01
            Sum of Squares    = .729365D+06
            R -squared        = .233117D+00

```

Variable	Coefficient	Standard Error	b/St.Er.	P[ Z >z]	Mean of X
X2	.2341713422	.71596797E -01	3.271	.0011	3.8895953
X3	.3703833036	.94402278E -01	3.923	.0001	4.2207458
X4	-.1090676626	.73213889E -01	-1.490	.1363	3.9716586
X7	.6669200976	.60069733E -01	11.102	.0000	3.7239903
X13	1.387746607	.97011341E -01	14.305	.0000	2.2173183

## Fixed Effects Panel Data Model 21

X19	4.937585087	.10343032	47.738	.0000	.86701895E	-01
X20	1.136524709	.11289247	10.067	.0000	.42022957E	-01
Constant	44.62217113	1.1804686	37.800	.0000		

```
--> REGRESS;Lhs=X22
      ;Rhs=X2,X3,X4,X7,X13,X19,X20
      ;Str=X23
      ;Panel
      ;Output=2
      ;Wts=X16
      ;Fixed $
```

```
+-----+
| OLS Without Group Dummy Variables
| Ordinary least squares regression Weighting variable = X16
| Dep. var. = X22 Mean= 52.55493863 , S.D.= 9.343603189
| Model size: Observations = 10895, Parameters = 8, Deg.Fr.= 10887
| Residuals: Sum of squares = 731652.8662 , Std.Dev.= 8.19782
| Fit: R-squared= .230712, Adjusted R-squared = .23022
| Model test: F[ 7, 10887] = 466.44, Prob value = .00000
| Diagnostic: Log-L = -38376.9800, Restricted(b=0) Log -L = -39805.8044
| LogAmemiyaPrCrt.= 4.208, Akaike Info. Crt.= 7.046
| Panel Data Analysis of X22 [ONE way]
| Unconditional ANOVA (No regressors)
| Source Variation Deg. Free. Mean Square
| Between 20751.0 3. 6917.00
| Residual 930327. 10891. 85.4216
| Total 951078. 10894. 87.302 9
+-----+
```

Variable	Coefficient	Standard Error	b/St.Er.	P[ Z >z]	Mean of X
X2	.2150986215	.72295926E -01	2.975	.0029	3.8895953
X3	.3746630995	.95235646E -01	3.934	.0001	4.2207458
X4	-.1516385665	.73563678E -01	-2.061	.0393	3.9716586
X7	.7025006784	.59425942E -01	11.821	.0000	3.7239903
X13	1.398697309	.97973819E -01	14.276	.0000	2.2173183
X19	5.040076855	.10413200	48.401	.0000	.86701895E -01
X20	1.035652953	.11377009	9.103	.0000	.42022957E -01
Constant	44.54122268	.41506161	107.312	.0000	

```
+-----+
| Least Squares with Group Dummy Variables
| Ordinary least squares regression Weighting variable = X16
| Dep. var. = X22 Mean= 52.55493863 , S.D.= 9.343603189
| Model size: Observations = 10895, Parameters = 11, Deg.Fr.= 10884
| Residuals: Sum of squares= 716924.2 581 , Std.Dev.= 8.11601
| Fit: R-squared= .246198, Adjusted R-squared = .24551
| Model test: F[ 10, 10884] = 355.48, Prob value = .00000
| Diagnostic: Log-L = -38266.1998, Restricted(b=0) Log -L = -39805.8044
| LogAmemiyaPrCrt.= 4.189, Akaike Info. Crt.= 7.027
| Estd. Autocorrelation of e(i,t) -.000540
+-----+
```

Variable	Coefficient	Standard Error	b/St.Er.	P[ Z >z]	Mean of X
----------	-------------	----------------	----------	----------	-----------

## Fixed Effects Panel Data Model 22

X2	.2342486655	.71596894E -01	3.272	.0011	3.8895 953
X3	.3704126069	.94402933E -01	3.924	.0001	4.2207458
X4	-.1089542352	.73215840E -01	-1.488	.1367	3.9716586
X7	.6668605394	.60076332E -01	11.100	.0000	3.7239903
X13	1.387686017	.97011428E -01	14.304	.0000	2.2173183
X19	4.937094032	.10343209	47.733	.0000	.86701895E -01
X20	1.136968803	.11289363	10.071	.0000	.42022957E -01

## Estimated Fixed Effects

Group	Coefficient	Standard E rror	t-ratio
1	45.84312	.42968	106.69106
2	45.48305	.44005	103.35884
3	43.06207	.42588	101.11422
4	44.10279	.44472	99.16970

## Test Statistics for the Classical Model

Model	Log -Likelihood	Sum of Squares	R -squared
(1) Constant term only	-39805.80420	.9510780165D+06	.0000000
(2) Group effects only	-39685.63264	.9303270109D+06	.0218184
(3) X - variables only	-38376.97986	.7316528662D+06	.2307120
(4) X and group effects	-38266.19962	.7169242581D+06	.2461983

## Hypothesis Tests

Likelihood Ratio Test				F Tests		
	Chi-squared	d.f.	Prob.	F	num. denom.	Prob value
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(3) vs (1)	2857.649	7	.00000	466.435	7 10887	.00000
(4) vs (1)	3079.209	10	.00000	355.481	10 10884	.00000
(4) vs (2)	2838.866	7	.00000	462.825	7 10884	.00000
(4) vs (3)	221.560	3	.00000	74.53 4	3 10884	.00000



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